Exercises 2.3

[11-12-05-MT23]

1. $\sin \frac{\pi}{12}$ 2. $\sin \frac{5\pi}{12}$ 3. $\cos \frac{5\pi}{12}$ 4. $\sin \frac{11\pi}{12}$
5. $\cos \frac{11\pi}{12}$ 6. $\cos \frac{7\pi}{12}$ 7. $\sin \frac{7\pi}{12}$

Verify that each of the following equations is an identity.

8. $\cos\left(\frac{\pi}{6} + x\right) \equiv \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x.$
9. $\sin\left(x + \frac{\pi}{4}\right) \equiv \frac{\sqrt{2}}{2}(\sin x + \cos x).$
10. $\sin\left(\theta + \frac{\pi}{2}\right) \equiv \cos\theta.$
11. $\cos\left(\theta + \frac{\pi}{2}\right) \equiv -\sin\theta.$
12. $\sin(\theta + \pi) \equiv -\sin\theta.$
13. $\cos(\theta + \pi) \equiv -\cos\theta.$
14. $\cos 4\theta \equiv \cos 3\theta \cos\theta - \sin 3\theta \sin\theta.$
15. $\sin 7x \equiv \sin 4x \cos 3x + \cos 4x \sin 3x.$
16. $\sin 3\phi \equiv \sin 5\phi \cos 2\phi - \cos 5\phi \sin 2\phi.$
17. $\cos 5y \equiv \cos 9y \cos 4y + \sin 9y \sin 4y.$
18. $\sin(x + \pi) - \sin(x - \pi) \equiv 0.$
19. $(\sin x + \cos\beta)^2 + (\cos x + \sin\beta)^2 \equiv 2[\sin(x + \beta) + 1].$
20. $1 - \tan\theta\tan\phi \equiv \frac{\cos(\theta + \phi)}{\cos\theta\cos\phi}.$
21. $\cos(\alpha + \beta)\cos(\alpha - \beta) \equiv (\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2.$
22. $(\cos\alpha\cos\beta)^2 - (\sin\alpha\sin\beta)^2 \equiv \cos^2\alpha - \sin^2\beta.$
23. $\tan(\alpha + \beta) \equiv \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ [Use $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.]
24. $\cot(\theta + \phi) \equiv \frac{\cot\theta\cot\phi - 1}{\cot\phi + \cot\theta}.$
25. $\sec(x + y) \equiv \frac{\sec x \sec y}{1 - \tan x \tan y}.$
26. $\csc(\alpha + \beta) \equiv \frac{\csc\alpha\csc\beta}{\cot\beta + \cot\alpha}.$
27. $\tan(\theta - \phi) \equiv \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}.$
28. $\cot(x - y) \equiv \frac{\cot x \cot y + 1}{\cot y - \cot x}.$
29. $\sec(\alpha - \beta) \equiv \frac{\sec\alpha\sec\beta}{1 + \tan\alpha\tan\beta}.$
30. $\csc(\theta - \phi) \equiv \frac{\csc\theta\csc\phi}{\cot\phi - \cot\theta}.$
31. $\tan\left(\theta + \frac{\pi}{2}\right) \equiv -\cot\theta.$
32. $\cot\left(\alpha + \frac{\pi}{2}\right) \equiv -\tan\alpha.$
33. $\sin 2\phi \equiv 2\sin\phi\cos\phi.$
34. $\cos 2x \equiv \cos^2 x - \sin^2 x.$
35. $\cos 2y \equiv 2\cos^2 y - 1.$
36. $\cos 2z \equiv 1 - 2\sin^2 z.$
37. $\tan 2\alpha \equiv \frac{2\tan\alpha}{1 - \tan^2\alpha}.$
38. $\tan(\alpha + \beta) \equiv \frac{\cot\beta + \cot\alpha}{\cot\alpha\cot\beta - 1}.$
39. $(\sin x \cos y)^2 - (\cos x \sin y)^2 \equiv \sin^2 x - \sin^2 y.$
40. $\sin(x + y)\sin(x - y) \equiv \sin^2 x - \sin^2 y.$

Exercises 2.3

1. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

3. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

5. $-\dfrac{\sqrt{2}+\sqrt{6}}{4}$

7. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

16. $$\begin{aligned}\sin 3\phi &= \sin(5\phi - 2\phi)\\ &= \sin 5\phi \cos 2\phi - \cos 5\phi \sin 2\phi.\end{aligned}$$

20. $$\begin{aligned}\frac{\cos(\theta+\phi)}{\cos\theta\cos\phi} &= \frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi}\\ &= \frac{\cos\theta\cos\phi}{\cos\theta\cos\phi} - \frac{\sin\theta\sin\phi}{\cos\theta\cos\phi}\\ &= 1 - \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\phi}{\cos\phi}\right)\\ &= 1 - \tan\theta\tan\phi.\end{aligned}$$

25. $$\begin{aligned}\sec(x+y) &= \frac{1}{\cos(x+y)}\\ &= \frac{1}{\cos x\cos y - \sin x\sin y}\\ &= \frac{\dfrac{1}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y} - \dfrac{\sin x\sin y}{\cos x\cos y}}\\ &= \frac{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{1}{\cos y}\right)}{1 - \left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{\sin y}{\cos y}\right)}\\ &= \frac{\sec x\sec y}{1 - \tan x\tan y}\end{aligned}$$

39. $$\begin{aligned}(\sin x\cos y)^2 - (\cos x\sin y)^2 &= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y\\ &= \sin^2 x\cos^2 y + \sin^2 x\sin^2 y\\ &\quad -\sin^2 x\sin^2 y - \cos^2 x\sin^2 y\\ &= \sin^2 x(\cos^2 y + \sin^2 y)\\ &\quad - (\sin^2 x + \cos^2 x)\sin^2 y\\ &= \sin^2 x(1) - (1)\sin^2 y\\ &= \sin^2 x - \sin^2 y.\end{aligned}$$